On Exact Polya, Hilbert-Artin and Putinar's Representations

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We consider the problem of finding exact sums of squares (SOS) decompositions for some classes of non-negative multivariate polynomials, relying on semidefinite programming (SDP) solvers.

We start by providing a hybrid numeric-symbolic algorithm computing exact rational SOS decompositions for polynomials lying in the interior of the SOS cone. It computes an approximate SOS decomposition for a perturbation of the input polynomial with an arbitrary-precision SDP solver. An exact SOS decomposition is obtained thanks to the perturbation terms. We prove that bit complexity estimates on output size and runtime are both polynomial in the degree of the input polynomial and simply exponential in the number of variables. Next, we apply this algorithm to compute exact Polya, Hilbert-Artin and Putinar's representations for positive definite forms and positive polynomials over basic compact semi-algebraic sets.

Our algorithms are implemented in the Maple package RealCertify [4]. We compare the performance of RealCertify with existing methods in computer algebra, including cylindrical algebraic decomposition [2] and critical point method [1, 3]. These results have been recently published in [5] and [6].

Références

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