Reduction operators and completion of linear rewriting systems

Cyrille Chenavier[1]

[1]INRIA Lille - Nord Europe

Rewriting theory provides a framework for studying algebraic structures, such as rings or modules, by mean of oriented equalities. In particular, fundamental properties, such as termination and confluence, guarantee that canonical forms are computed by applying reduction steps as long as it is possible.

An alternative approach, initialised by Bergman [1] in the context of noncommutative Gröbner bases, consists in representing reduction steps by linear maps, called *reduction operators*. The method works as follows : consider an alphabet X together with a monomial order $\preceq$ over the set of words X^*, as well as a set of non commutative polynomials $f_1, \cdots, f_r \in \mathbb{K}X^*$. One assigns to this data the endomorphisms $T_1, \cdots, T_r$ of $\mathbb{K}X^*$ defined by $T_i(w\mathrm{lm}\,(f_i)\,w') = w\,(\mathrm{lm}\,(f_i) - f_i)\,w'$ for every words w and w'. Such defined, these endomorphisms model all the possible reduction steps defined from the f_i's.

The endomorphisms T_i defined in the previous paragraph satisfy the following decreasing property : for every word w, we have $\mathrm{lm}\,(T_i(w)) \preceq \mathrm{lm}\,(w)$. More generally, a reduction operator relative to an ordered set $(G, \preceq)$ is an endomorphism T of the vector space spanned by G satisfying $\mathrm{lm}\,(T(g)) \preceq \mathrm{lm}\,(g)$ for every $g \in G$.

Based on the work [2], the goal of this talk is to present the general theory of reduction operators. In particular, we equip the set of reduction operators with a lattice structure in terms of kernels. We deduce from this structure, lattice interpretations of the confluence property and of the completion procedure for constructing confluent rewriting systems.

Références

[1] George M. Bergman. The diamond lemma for ring theory. *Adv. in Math.*, 29(2) :178–218, 1978.

[2] Cyrille Chenavier. Reduction operators and completion of rewriting systems. *J. Symbolic Comput.*, 84 :57–83, 2018.